ON COMPUTATIONS FOR THE MAYA CALENDAR

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IN *A Method which may have been used by the Mayas in Calculating Time*, and also in *The Numeration, Calendar Systems, and Astronomical Knowledge of the Mayas*, Appendix VII, Mr. C. P. Bowditch gave a rule for shortening the computations for finding a terminal date in the Maya calendar count when the initial date and the interval of time elapsed from it are given. This rule has perhaps not had the use it deserves, partly because of the natural prejudice of modern scholars in favor of expressing numbers in the familiar decimal system, and partly because the rule as given by Mr. Bowditch is still a little cumbrous. It is the purpose of the present paper, first, to develop modifications of Mr. Bowditch's method which make it more easily used, and second, to give a new formula for the solution of the reverse problem, namely, given two dates to find the interval of time between them. These deductions will be prefaced by some general considerations concerning computations in the Maya system of numeration. Their system, so far as we know it, was a curious mixture. The bar and dot notation is quinary, the face numerals from 13 (or sometimes 14) to 19 and the corresponding native names for these numbers are decimal, and the calendar system as a whole is vigesimal, with the exception of the step from uinals to tuns. Irregular though the system is, it is the author's belief that the greatest facility in handling Maya time periods will come by keeping them expressed so far as possible in the Maya fashion.

Some operations present no difficulty in an irregular system like the Maya. We handle repeatedly somewhat similar notations. For instance 4 hours 3 minutes 6 seconds; £12 13 s. 6 d.; 78° 24′ 30″ and so on. There is no difficulty in performing simple operations on these. Adding them, subtracting them, multiplying or dividing them by small numbers are all quite easy, the only pre-

caution to be observed is the reduction of one kind of unit to the next in carrying or borrowing. So with the Maya system of time periods. For instance:

$$\begin{array}{r} 17.\ 7.\ 6.12.\ 1 \\ +\quad 4.15.\ 7.10.13 \\ \hline 1.\ 2.\ 2.14.\ 4.14 \end{array} \qquad\qquad \begin{array}{r} 3.\ 1.14.\ 6.\ 8 \\ -\quad 1.\ 2.\ 6.17.\ 9 \\ \hline 1.19.\ 7.\ 6.19 \end{array}$$

The only point requiring special care here is to note that the amount carried from uinals to tuns or borrowed from tuns to uinals is 18. Again as an example of multiplication by a small number let us find 11 times 2.12.13. 0.

$$\begin{array}{r} 2.12.13.0 \\ 11 \\ \hline 1.\ 8.19.17.0 \end{array}$$

The steps are: $11 \times 0 = 0$. $11 \times 13 = 143$ which is (dividing by 18) 7 tuns and 17 uinals. Write the 17 uinals and carry 7 tuns. $11 \times 12 = 132$ and 7 to carry is 139 which is (dividing by 20) 19 and 6 to carry. $11 \times 2 = 22$ and the 6 makes 28, which is 8 and 1 to carry. Write the 1. Short division goes in a very similar way: to divide 14. 4.17. 3. 2. by 5:

$$\begin{array}{l} \underline{5)\ 14.\ 4.17.\ 3.\ 2} \\ \quad\ \ 2.16.19.\ 7.16 \text{ and a remainder of 2.} \end{array}$$

The steps are: 5 into 14 twice and 4 over; the 4 over is 4 twenties or 80 of the next grade, a total of 84 of the next grade; 5 into 84 16 times and 4 over, which makes a total of 97 of the next grade; 5 into 97 19 times and 2 over. 2×18 is 36 of the next grade and 3 makes a total of 39. 5 into 39 7 times and 4 over, 82 of the next grade. 5 into 82 16 times and a remainder of 2.

However, in the case of a multiplication by a larger number, so that handling it would require the use of partial products, our ordinary process breaks down. Suppose we try to multiply in the usual way, by means of partial products, 2.12.13.0 by 73. We can multiply by the 3 at once

$$\begin{array}{r} 2.12.13.0 \\ 3 \\ \hline 7.18.\ 3.0 \end{array}$$

but the next partial product cannot be formed by multiplying by 7 and setting over one place, because the actual step is multiplying by 70 and the setting over in the Maya system is not equivalent to multiplying by 10 but by 20 in most cases, but to multiplying by 18 when setting across the 18 step from uinals to tuns. If it were not for this step, that is, if the Mayas used a purely vigesimal system, writing the multiplier also in the same system would solve the difficulty and allow us to apply our ordinary processes at once. A long division involves fully as many difficulties for a similar reason. This does not mean that these processes could not be carried out by some means, but that it is impractical because our usual rules do not apply to the operations. Fortunately, however, the calculations ordinarily required for the Maya calendar are of a very restricted variety, and by the use of special methods for these problems nearly all time units may be kept in the Maya system, avoiding the tedious and rather useless reduction to the decimal notation that has proven the popular method with most investigators (*e. g.*, Förstemann, Seler, S. G. Morley).

There are two principal problems that we have to solve in connection with the calendar: (1) Given a starting date by day coefficient, day name, month coefficient, and month name, and a period of time counted from it, required the terminal date by coefficients and names. (2) Given the two dates, required the distance from one to the other. Let us take them up in order.

As is explained in treatises on the Maya calendar finding one date a certain distance from another is unaffected by diminishing the distance by any number of complete calendar rounds of 52 haab or years. To subtract the greatest possible number of complete rounds diminishes the subsequent work somewhat.[1] 1 haab = 1.0.5. Multiply this by 52 as follows: 52 × 5 kins = 260 kins or 13 uinals; 52 × 1 tun = 52 tuns = 2 katuns and 12 tuns. Hence 1.0.5 × 52 = 2.12.13.0. That is 1 calendar round = 2.12.-13.0. For short intervals, say less than 10 rounds, it is easy to multiply this at once for the purpose of getting the multiple to

[1] This process of diminishing the distance by calendar rounds, while the usual method, can be avoided, as will be shown later.

be subtracted. *E. g.*, 7 rounds = 7 × 2.12.13.0 = 18.9.1.0. The process is: 7 × 0 = 0, 7 × 13 = 91 = 18 × 5 + 1, write 1 and carry 5; 7 × 9 = 84, and 5 to carry is 89 which = 4 × 20 + 9, write 9 and carry 4; 7 × 2 = 14, add 4 = 18. But for long intervals such as are found in the Initial Series this is too difficult. A table is therefore desirable. Such a table may be found in Morley, *Bureau of American Ethnology*, Bulletin 57, p. 144. The column headed Cycles, etc., is the one to use. When all complete calendar rounds have been subtracted the distance number to be treated is always less than 2.12.13.0.

The problem of finding the terminal date consists of three parts: (1) Counting from the given day name a number equal to the remainder on dividing the distance by 20 to find the new day name; (2) Counting from the day coefficient a number equal to the remainder on dividing the distance by 13; (3) Counting from the day in the year a number equal to the remainder on dividing the distance by 365 days or 1.0.5.

1. There is no difficulty with the first of these. The required number is exactly the kin number of the given distance, since 20 kins = 1 uinal, and the tuns, etc., are all exact multiples of the uinal. To get the day name count forward (backward sometimes when the count as a whole is backward. I shall hereafter assume that it is forward. The methods for finding the remainders apply) a number equal to the kin number.

2. The remainder on dividing by 13 might be found by the process indicated above for short division, but the following modification of the method of Mr. Bowditch already referred to is practically much more convenient. To understand it we shall first develop some general principles.

A. The remainder on dividing a number by a divisor is not affected by adding to or subtracting from the number any exact multiple of the divisor. For, regard the process of dividing as counting around and around in a circle composed of as many places as there are units in the divisor (of course it is from just such circular counting in the Maya calendar that the need for the remainder arises). Then it is clear that to add or subtract an

exact multiple of the divisor will change the number of complete rounds but will not affect at all the amount left over. If a number be diminished by the next less exact multiple of the divisor, that is if the number be replaced by the remainder on dividing by the divisor, the process is frequently referred to as "casting out" the divisor. Thus, casting out 13 from 68 gives 3; casting out 13 from 97 gives 6, etc.

B. If a number divided by a divisor gives a certain remainder, a multiple n times the number will have a remainder n times as great as the original remainder, or will differ from it by a multiple of the divisor. Thus, $12/9$ gives a remainder 3, hence 2×12 divided by 9 gives a remainder $2 \times 3 = 6$. Again, 5×12 divided by 9 gives a remainder 6 which $= 5 \times 3 - 9$. That this must be the case will appear from considering again the counting around a circle. For if the number once counted around and around ends a certain distance (*i. e.*, the remainder) beyond the starting point, counted twice it will end as much further beyond the new starting point, *i. e.*, twice as far from the first starting point, and so on. If these together make up more than a complete revolution it will be necessary to deduct a multiple of the divisor. Further examples follow: $20/13$ leaves a remainder of 7; therefore 4×20, that is 80, leaves a remainder of $7 \times 4 - 2 \times 13 = 2$ as may readily be verified. $360 = 20 \times 18$. Hence $360/13$ gives remainder of $7 \times 18 = 126$ less 117 (13×9) or 9. This last is more easily done by the aid of Principle *A*, thus: $20 \times 18 = 20 \times 13 + 20 \times 5$. Casting out 20×13 (Prin. *A*) we may treat 20×5 instead. $5 \times 7 = 35$; casting out 13, $35 - 26 = 9$ as before. In using Principle *B*, therefore it is never necessary to use a multiplier larger than the divisor, in these cases 13. If one such occurs the remainder is unaltered by casting out 13 from it at once.

We will now apply these principles to finding the remainder on dividing 1 uinal, 1 tun, 1 katun, and 1 cycle (the cycle is unnecessary if all complete calendar rounds have been subtracted) respectively by 13.

1 uinal divided by 13 remainder 7.

1 tun = 18 uinals which will have same remainder as $18 - 13 = 5$ then $7 \times 5 = 35$ cast out 13 remainder 9.

1 katun = 20 tuns, same remainder as 7 tuns; then $7 \times 9 = 63$ cast out 13 remainder 11.

1 cycle = 20 katuns, same remainder as 7 katuns, then $7 \times 11 = 77$ cast out 13 remainder 12.

To put these results together we need another principle, *C*. Namely that the sum of the remainders of several numbers each divided by a divisor, after casting out the divisor from this sum, is the same as the remainder on dividing the sum of the numbers themselves.

Another consideration of counting in a circle will show the truth of this.

If then we have a number expressed in cycles, katuns, uinals, and kins the total remainder is given by the following sum

kin number + uinal number × 7 (7 is remainder on dividing 1 uinal by 13, then apply Principle *B*),
+ tun number × 9 (9 is remainder from 1 tun as found on p. 53, then by Principle *B*),
+ katun number × 11 (11 is remainder from 1 katun as found above),
+ cycle number × 12 (12 is remainder from 1 cycle as found above).

In using this method 13 should be cast out at every possible stage. If the kin number exceeds 13, cast it out. If the uinal, tun, katun, or cycle number exceeds 13 cast it out before forming the product, and cast out 13 from each product before adding it to the preceding and whenever the sum exceeds 13 cast it out.

This rule is quite feasible, but a further diminution of the size of the numbers involved is possible. Consider again counting around the divisor circle. If the remainder is more than half the divisor we shall evidently arrive at the same point more quickly by counting backward the difference between the remainder and a complete revolution (*i. e.*, the divisor). This difference represents the amount the number being counted falls short of the next larger multiple of the divisor. If we are considering multiples of the number, or sums of several numbers, their backward remainders

may be multiplied and added like ordinary remainders and the amount to be counted ahead finally found by deducting their total from the divisor or from the next greater multiple of the divisor.

Now it will be noticed that the remainders from 1 uinal, 1 tun, 1 katun and 1 cycle after dividing by 13, as found on pages 53–54, are all more than half of 13. The backward remainders, $13 - 7 = 6$, $13 - 9 = 4$, $13 - 11 = 2$, and $13 - 12 = 1$ will give us smaller multipliers. This suggests the following rule: Multiply the uinal number (first casting out 13 if possible) by 6 and cast out 13; multiply the tun number (less 13 if possible) by 4 and cast out 13; add this to the preceding result and so on. Then subtract this sum from 13, add the kin number (less 13 if possible) and again cast out 13. The order of these additions and subtractions may be interchanged with advantage giving the following final form for the rule:

RULE 1,[1] FOR FINDING THE REMAINDER ON DIVIDING BY 13 A PERIOD OF TIME EXPRESSED IN THE MAYA SYSTEM

Multiply the cycle number (less 13 if possible) by 1 (that is, take it as it stands).

Multiply the katun number (less 13 if possible) by 2. Add to preceding and cast out 13.

Multiply the tun number (less 13 if possible) by 4. Add to preceding and cast out 13.

Multiply the uinal number (less 13 if possible) by 6. Add to preceding and cast out 13.

Subtract this result from the kin number increased by 13 if necessary to keep the difference positive. This will give the remainder on dividing by 13.

[1] Whether the reader has followed the preceding reasoning or not it will prove a great time saver if he is to do much work with the Maya calendar to memorize this and the following rules. Its advantage consists largely in the fact that the small size of the numbers permits of performing all the operations mentally, or writing only the previous sum to be carried along. We often use rules without being very clear as to their reasons. For instance many people who cast out 9 in our decimal system by adding the digits could not justify the process. As a matter of fact the principles involved are similar to those here used. The sequence of multipliers in this rule (beginning with the cycle multiplier and ending with that for the uinals) 1, 2, 4, 6 is an easy one to remember. It is a curious coincidence that they add up to 13.

As already pointed out it is not *necessary* to handle distances as great as cycles, because the cycle number may always be removed by subtracting calendar rounds, but it is so easy to include them in the rule that it seems desirable to do so.

Some examples will now be treated. Required the remainder on dividing 9.6.4.10.5 by 13. Perform the operations thus: $9 \times 1 = 9$, $6 \times 2 = 12$, and $9 = 21$, cast out $13 = 8$. $4 \times 4 = 16$, and $8 = 24$, cast out $13 = 11$. $10 \times 6 = 60$, and $11 = 71$, cast out $13 = 6$. 5, the kin number, is less than 6, so add 13. $13 + 5 = 18$, less $6 = 12$, the required remainder.

Again, 9.17.15.16.11. $9 \times 1 = 9$. $17 - 13 = 4$, $4 \times 2 = 8$, $8 + 9 = 17$, $17 - 13 = 4$. $15 - 13 = 2$, $2 \times 4 = 8$, $8 + 4 = 12$. $16 - 13 = 3$, $3 \times 6 = 18$, $18 + 12 = 30$, $30 - 26 = 4$. 11 (kin number) $- 4 = 7$, the required remainder.

It will be noticed that the largest possible product with this method is $6 \times 12 = 72$, and the greatest possible multiple of 13 necessary for casting out therefore is $5 \times 13 = 65$. To carry out the whole process mentally involves no more labor than an ordinary short division.

The remainder so found is to be counted in the usual manner from the given day coefficient to find the new day coefficient.

3. Finding the remainder after dividing by 365, or a haab, or 1.0.5 may be done in a very similar way, except that the grade numbers cannot be diminished as before. The inconvenient form of the cycle remainder makes it perhaps a little easier to suppose the cycle number removed by deducting calendar rounds as previously explained but it is not necessary. Proceeding as for 13, the remainder for 1 kin is 1 kin, the remainder for 1 uinal is 20 kins or 1 uinal, the remainder for 1 tun is 360 kins or 18 uinals, but the backward remainder is much smaller, namely 5 kins. The backward remainder for 1 katun is 20 times this, or 5 uinals. In precisely the same way as before, in the case of 13, we get the following:

The improvements of the rule given here over Mr. Bowditch's method consist in casting out the 13 at every stage, thus bringing the work within the bounds of mental computation, in avoiding the plus and minus signs, which are confusing to a mind not mathematically trained, and in arranging the process in a rule consisting of very definite steps always to be taken in the same way.

RULE 2, FOR FINDING THE REMAINDER ON DIVIDING BY 1.0.5 A PERIOD OF TIME EXPRESSED IN THE MAYA SYSTEM

Multiply the tun number by 5 and write it as kins, reducing to uinals and kins if result exceeds 20.

Multiply the katun number by 5 and write it as uinals, reducing to uinals and tuns if result exceeds 18.

Add these two, and subtract their sum from the uinals and kins of the given distance, adding a sufficient multiple of 1.0.5. to keep the difference positive. The result will be the required remainder expressed in uinals and kins. It should be counted in this form, the uinal number being exactly adapted to counting uinals without further reduction.

This method may be extended to include cycles, thus obviating the necessity of subtracting calendar rounds by means of a table. The student may choose between the following and the calendar round table. The backward remainder from 1 katun was seen to be 5 uinals. From one cycle it is then 20 × 5 uinals or 100 uinals = 5.10.0. Casting out 5 × 1.0.5. this becomes 0.8.15. The remainder itself is then 1.0.5 − 8.15 = 9.10. The following addition therefore to the Rule just given takes care of cycles.

Add to the previous result the cycle number times 9.10 casting out 1.0.5. from the result. As the majority of cycle numbers is 9 the amount to be added in most cases is 9 × 9.10 = 4.13.10 which on casting out 4 × 1.0.5 gives 12.10. To remember this last figure and add it will probably save time over using the calendar round table.

Appendix to Rule 2.—Multiply 9.10 by the cycle number, add the previous result and cast out 1.0.5. If the cycle number is 9 add 12.10 at once (casting out 1.0.5. if possible).

Some examples will now be considered. Required the remainder on dividing 9.6.4.10.5 by 1.0.5.

$4 \times 5 = 20$ kins =	1.0	Given uinals and kins	10. 5
$6 \times 5 = 30$ uinals =	1.12.0	Multiple of 1.0.5 to keep result positive	2. 0.10
Sum	1.13.0	Sum	2.10.15
		Deduct	1.13. 0
			15.15
		Add 12.10 for the 9 cycles	12.10
		Sum	1.10. 5
		Cast out 1.0.5 for the required result	10. 0

The step marked with the brace would have been unnecessary if calendar rounds had been deducted at the start. In following the present method a change of order is perhaps better, as in the next example.

9.17.15.16.11

$15 \times 5 = 75$ kins =	3.15	Uinals and kins given	16.11
$17 \times 5 = 85$ uinals =	4.13. 0	Remainder for 9 cycles	12.10
Sum	4.16.15	Add $4 \times$ 1.0.5	4. 1. 0
		Sum	5.12. 1
		Deduct	4.16.15
		Required remainder	13. 6

Next let us apply these methods of finding remainders to a case from the Maya calendar. Required the terminal date a distance of 9.12.8.14.1 forward from 4 ahau 8 cumhu. The remainder on dividing by 20 is the kin number 1. The day name is therefore 1 forward from ahau; that is, imix. For the remainder on dividing by 13, $9 \times 1 = 9$. $12 \times 2 = 24$, $24 + 9 = 33$, $33 - 26 = 7$. $8 \times 4 = 32$, $32 + 7 = 39$, $39 - 39 = 0$. $14 - 13 = 1$, $1 \times 6 = 6$. $6 + 0 = 6$. 1 (kin number) $+ 13 = 14$, $14 - 6 = 8$. 8 days forward from 4 gives $4 + 8 = 12$. So far then 12 imix. For the remainder on dividing by 1.0.5.

$8 \times 5 = 40$ kins =	2.0	Uinals and kins given	14. 1
$12 \times 5 = 60$ uinals =	3.6.0	Remainder for 9 cycles	12.10
Sum	3.8.0	$2 \times$ 1.0.5 to make sum exceed 3.8.0	2. 0.10
		Sum	3. 9. 1
		Deduct	3. 8. 0
			1. 1

Count forward 1 uinal from 8 cumhu would be 8 pop, but we must deduct 5 kins as we pass uayeb, which gives 3 pop. Then 1 kin makes 4 pop. Result 12 imix 4 pop.

Again, let us reckon the terminal day reached by counting 9.0.19.2.4. from 4 ahau 8 cumhu. Counting forward the kin number, 4, from ahau the day name is evidently kan. Next find the remainder on dividing by 13. In writing the process the result of each step only will be written 9. 0, 9. $24 + 9 - 26 = 7$. $12 + 7 = 19$, *i. e.*, 6. 4 (kins) $+ 13 = 17$. $17 - 6 = 11$. 11 days forward from 4 is 2. So far 2 kan. $19 \times 5 = 95 = 4.15$.

Given................	2. 4
9 cycles..............	12.10
	14.14
Deduct..............	4.15
Result...............	9.19

Count forward 9 uinals from 8 cumhu, deducting 5 kins as you pass uayeb from the 19 kins, result 8 chen and 14 kins yet to go, which brings 2 yax. The whole result then is 2 kan 2 yax.

The Second Problem

Take up now the second problem stated above, namely, given two dates by names and coefficients, required the distance from the first to the second. It is evident that except for the fact that all complete calendar rounds must be thought of as rejected this problem is the reverse of the other. That is we must now find a distance such that the remainder on dividing by 20 (*i. e.*, the kin number) shall be what we get by counting from the first given day name to the other day name, and also such that the remainder on dividing by 13 shall be what we get by counting from the first given day coefficient to the other day coefficient, and also such that the remainder on dividing by 1.0.5. or 365 kins shall be what we get by counting from the first day in the year to the second. It will be convenient to use the following abbreviations:

For the count from the first day name to the second, X;
" " " " " " " coefficient to the second, Y;
" " " " " " " in the year to the second, Z.

Stated in terms of these symbols the problem is to find a distance such that the remainder on dividing by 20 (*i. e.*, the kin number) is X, on dividing by 13 is Y, and on dividing by 365 or 1.0.5 is Z. Let us try to satisfy the last two conditions first. If the distance divided by 13 leaves a remainder Y it must be possible to write it in the form $13m + Y$, where m is a whole number (the quotient). Similarly it can be written $365n + Z$, where n is a whole number. But these represent the same distance, hence $13m + Y = 365n + Z$ or dividing by 13

$$m = \frac{365n + Z - Y}{13}, \text{ which } = 28n + \frac{n + Z - Y}{13}.$$

Denote by $[Z - Y]_{13}$ the remainder on casting out 13 from $Z - Y$. Then the only part of this value of m which is not a whole number on the face of it is

$$\frac{n + [Z - Y]_{13}}{13}.$$

m however is a whole number, consequently the numerator of this must be divisible by 13. This will happen if $n + [Z - Y]_{13} = 13$ or 26 or 39 or 52, etc. Then n must be $13 - [Z - Y]_{13}$ or this number increased by a multiple of 13. The distance therefore is $365 \times \{13 - [Z - Y]_{13}\} + Z$ or this plus 365×13 or $365 \times 13 \times 2$ or $365 \times 13 \times 3$. It is not necessary to go further, for $365 \times 13 \times 4$ is a complete calendar round. What determines the proper multiple of 365×13 to add? Evidently the one remaining condition to be satisfied, that the kin number of the found distance shall equal X. Expressing distances now in the Maya notation the rule may be formulated thus:

Rule 3, for Finding the Distance between Two Given Dates

Compute $\{13 - [Z - Y]_{13}\} \times 1.0.5 + Z$,[1] where Y is the count from one day coefficient to the other, and Z is the count from one day in the year to the other, and $[Z - Y]_{13}$ signifies the remainder on casting out 13 from $Z - Y$. If this result has the correct kin

[1] The division by 13 in finding $[Z - Y]_{13}$ should be done in the Maya notation by the short division process explained on page 50.

number (*i. e.*, X, the count from the first day name to the second) it is the required distance. If not, subtract its kin number from the required one, X (borrowing 20 if necessary). The difference must be 5, 10, or 15.

$$\left.\begin{array}{lll} \text{If } 5\ (= 1 \times 5) & \text{add} & 1 \times 13 \times 1.0.5 = 13.3.5 \\ \text{“ } 10\ (= 2 \times 5) & \text{“} & 2 \times 13 \times 1.0.5 = 26.6.10 \\ \text{“ } 15\ (= 3 \times 5) & \text{“} & 3 \times 13 \times 1.0.5 = 39.9.15 \end{array}\right\}^{1}$$

This will give the required distance. If Z should be less than Y it may be increased by 1.0.5 in computing $[Z - Y]$ so as to keep this difference positive.

To facilitate the use of this method it will now be given in a less exact way which is perhaps easier to remember and which is enough to recall the successive steps.

RULE 3*a*. The Year Position Difference Less the Day Coefficient Difference. Cast out 13. Subtract from 13. Multiply by a year, and add to the year position difference. If this is not right add enough 13 years to make it right. Test correctness by comparing kin number with day name difference.

As an example of the application of this rule let us find the distance from 2 kan 2 yax to 7 muluc 17 tzec. Here X = count from kan to muluc = 5. $Y = 7 - 2 = 5$. Z = count from 2 yax to 17 tzec = 13 uinals + 17 − 2 kins + 5 kins in uayeb = 14.0. The formula to be computed is then $\{13 - [14.0 - 0.5]_{13}\} \times 1.0.5 + 14.0$. Now $14.0 - 0.5 = 13.15$. $[13.15]_{13} = [15]_{13} = 2$. $13 - 2 = 11$. $11 \times 1.0.5 = 11.2.15$.[2] $11.2.15 + 14.0 = 11.16.15$. But the day name difference, X, is 5 which does not agree with this 15 kin number. Now $5 + 20 - 15 = 10$, which is 2 times 5. Add therefore 2 times $13 \times 1.0.5 = 26.6.10$ and the result is $38.5.5 = 1.18.5.5$, which is the required distance.

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[1] Of course 26 tuns and 39 tuns should be reduced to katuns and tuns to adhere to the Maya notation, but it is probably easier to remember them as written here, by means of the progressions, 5, 10, 15 for kins; 3, 6, 9 for uinals; and 13, 26, 39 for tuns.

[2] Multiplication of this product should be carried out in the Maya notation as explained on page 50.